Quantum decoherence, charge detection, and single-electron entanglement in a mesoscopic ring

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We investigate the effect of local charge detection in an Aharonov-Bohm ring composed of N identical coupled quantum dots. It is found that the single-electron persistent current of the ring is not fully suppressed in the limit of perfect charge detection except for the case of $N=2$. This property is analyzed for a single electron in the number-state representation. We point out that the nonvanishing persistent current can be understood in terms of the single-electron entanglement in the number-state representation. In addition, we find that the local charge detection may produce a finite persistent current even in the absence of the external magnetic fields.

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I. INTRODUCTION

Quantum decoherence, that is, the emergence of a particlelike behavior in quantum theory, is a fundamental aspect in understanding the crossover between the quantum and the classical world.[1] For instance, suppression of interference for a single particle passing through a two-path interferometer coupled to its environment[2] can be understood either in terms (i) of path information transferred to the environment or (ii) of the phase uncertainty resulting from the back action caused by the environment. These two different interpretations are mathematically equivalent.[2] It has been widely accepted that the decoherence is inevitably caused by the "disturbance" during the detection process of the environment.

Controlled decoherence in a two-path interferometer is a nice way to investigate the origin of quantum decoherence. It has been realized in two-path interferometers with photons,[3] atoms,[4] and electrons in solid state.[5] For an electronic which-path interferometer,[5] Aharonov-Bohm (AB) interferometry has been used with a quantum dot inserted in an arm of the two paths while the path detection could be performed by a quantum point-contact charge detector nearby the quantum dot. Recently, solid-state interferometry has enabled investigation of this issue by highlighting the role of the path information.[6,7] By adopting a closed-loop AB interferometer instead of the two paths, it has been confirmed that the role of information acquisition may be more important than the "disturbance" caused by the detector.[6,7]

This electronic quantum decoherence has mostly been investigated in transport of particles through interferometers: basically a nonequilibrium phenomenon. On the other hand, what happens for a system at equilibrium would be an interesting question.[8] A good example is a mesoscopic ring with its size within the phase coherence length.[9,10] In this case an external magnetic flux induces a circulating persistent current to the extent that the quantum coherence is preserved. Various interactions with the environment may induce decoherence and suppress the persistent current in the mesoscopic ring.

In this study, we introduce a controllable decoherence of the ring by considering a local charge detector. The Aharonov-Bohm ring is composed of N tunnel-coupled identical quantum dots. We limit our study to a single electron in the ring, which is enough to discuss our main observation. To describe the decoherence source in the ring, we consider a local charge detector nearby to one of the dots (see Fig. 1). The effect of charge detection is formally introduced through the entanglement between the charge state of a quantum dot and the detector state. Our main observations are that (1) the persistent current in the ring is not fully suppressed in the limit of perfect local charge detection, (2) the behavior of the persistent current is well described in terms of the entanglement of a single electron in the number-state representation, and (3) the nonvanishing persistent current under perfect charge detection can also be understood through the number-state entanglement.

This paper is organized as follows. In Sec. II, we introduce a mesoscopic ring which consists of N identical quantum dots. We show that the ground state of the ring for a single electron is the entangled W state[11,12] in the number-state representation. We consider the local charge detection on one of the quantum dots in Sec. III. Within the formal approach of local charge detection, the reduced density matrix for the ground state is written in terms of the detection parameter which represents the strength of the detection. In Sec. IV, the effect of local charge detection on the persistent current is discussed. It is shown that local charge detection can induce a persistent current without applying magnetic

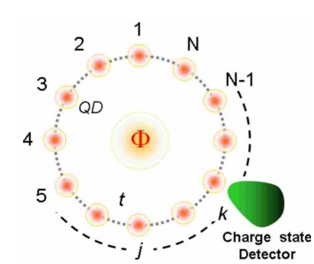

FIG. 1. (Color online) An Aharonov-Bohm ring composed of N-identical quantum dots. Each quantum dot has a single energy level. The dot is empty or singly occupied. The electron can move from a dot to its neighbor dots with tunneling amplitude t. The Aharonov-Bohm flux Φ penetrates the ring and induces the persistent current. A local charge detector is placed near the kth quantum dot and detects the charge of the kth quantum dot.

fields. Section V is devoted to the discussion of the persistent current and the effect of the local charge detection in terms of the single-electron entanglement. Also, we address the effect of multiple charge detectors. Possible experimental realization is briefly discussed in Sec. VI. Finally, the conclusions are given in Sec. VII.

II. MODEL

The model system under consideration is schematically drawn in Fig. 1. The N quantum dots are assumed to have the identical single energy levels and the identical tunnel coupling with its nearest-neighbor quantum dots. This condition can be achieved by controlling the parameters in the experiment. An external AB flux Φ threads the ring and is taken into account in the phase factor of tunnel coupling. The jth dot ($j=1,2,\ldots,N$) is either empty or singly occupied (denoted as $|0\rangle_j$ and $|1\rangle_j$, respectively, in the number-state representation). With this number-state representation, the ring Hamiltonian is written as

$$H = -t\sum_{j=1}^{N}\left(e^{2\pi i\phi/N}|j+1\rangle\langle j| + e^{-2\pi i\phi/N}|j\rangle\langle j+1|\right), \qquad (1)$$

where t is the hopping matrix element, and $|j\rangle = |0\rangle_1|0\rangle_2\cdots|1\rangle_j\cdots|0\rangle_N$ corresponds to the state that the electron is in the jth dot. The periodic boundary condition $|N+1\rangle=|1\rangle$ is imposed to describe the ring. The phase factor $\phi=\Phi/\Phi_0$ comes from the Aharonov-Bohm flux Φ piercing the ring where $\Phi_0=hc/e$ is the unit flux quantum.

The ground state of Hamiltonian (1) has energy $E_N=-2t\cos[2\pi\phi/N]$ and the wave function of the form (for $-1/2<\phi<1/2$)

$$|\Psi_G\rangle = \sum_{j=1}^{N}\frac{1}{\sqrt{N}}|j\rangle = \frac{1}{\sqrt{N}}(|100\cdots\rangle + |010\cdots\rangle + \cdots|00\cdots1\rangle). \qquad (2)$$

It is interesting to note that this is a single-electron-entangled state. Further, it is equivalent to the W state[11,12] of the N qubits (denoted as $|W_N\rangle$ in the following) in the number-state representation.

About single-particle entanglement, ambiguous views on the nature of "particle" have raised a debate based on the argument that at least two particles are needed to generate an entangled state.[13] However, a recent theoretical study has shown that nonlocal correlations of single particles can exhibit in a delocalized state.[14] Further, its agreement with recent experiments[15] has verified the nonlocal nature of a single-particle-entangled state. Aside from this subtle issue, we find it useful to introduce the single-particle entanglement for making a quantitative relation between the degree of entanglement and the amplitude of persistent current.

III. LOCAL CHARGE DETECTOR AND THE REDUCED DENSITY MATRIX OF THE RING ELECTRON

Suppose that the charge detector is sensitive to the charge state of the kth dot in the ring, as illustrated in Fig. 1. The mesoscopic ring may lose its coherence by a local charge detection on one of the quantum dots. In other words, the information of charge states of the dot is transferred to the local charge detector during the charge detection process and, in turn, the phase coherence of the dot charge state is lost. Instead of introducing a specific interaction between the ring and the local charge detector, we employ a (nonspecific model) formal approach to describe the decoherence (information transfer) of the ring. This approach allows us to capture an essential physics, even though in an experimental situation the detector states are rather complex and involve part of the environment surrounding the dot as well.

Before the charge detector obtains the charge information of the kth quantum dot, the composite system consisting of the ring and the detector is described by a direct product

$$|\Psi_{\text{tot}}\rangle = |\Psi_G\rangle \otimes |d\rangle, \qquad (3)$$

where $|d\rangle$ is the detector state. The interaction between the charge detector and the dot results in an evolution of the composite system. When the charge detector detects the charge state of the kth dot in the ring, the two subsystems get entangled given in the form

$$|\Psi_{\text{tot}}\rangle = \sqrt{\frac{N-1}{N}}|W_{N-1}^{(k)}\rangle \otimes |d_0\rangle + \sqrt{\frac{1}{N}}|k\rangle \otimes |d_1\rangle, \qquad (4)$$

where $|d_0\rangle$ ($|d_1\rangle$) represents the detector state when the kth dot is empty (occupied). $|W_{N-1}^{(k)}\rangle = \Sigma_{j=1(j\neq k)}^{N} 1/\sqrt{N-1}|j\rangle$ is the W state of the $N-1$ quantum dots with the kth dot excluded. Actually, the states d_0 and d_1 can formally contain anything other than the orbital degree of freedom of the ring electron. In our study, the complexity of the detector states does not produce any difficulties because the detector affects the physical properties of the ring only through the overlap integral $\delta=\langle d_0|d_1\rangle$.

The reduced density matrix of the ring is obtained by making a partial trace of the total density matrix $\varrho_{\text{tot}} = |\Psi_{\text{tot}}\rangle\langle\Psi_{\text{tot}}|$ over the charge detector degree of freedom,

$$\begin{aligned}\varrho_N(\delta) &= \text{Tr}_{\text{det}}[\varrho_{\text{tot}}] \\ &= \frac{N-1}{N}|W_{N-1}^{(k)}\rangle\langle W_{N-1}^{(k)}| + \frac{1}{N}|k\rangle\langle k| + \delta\frac{\sqrt{N-1}}{N}|k\rangle\langle W_{N-1}^{(k)}| \\ &\quad + \text{H.c.}\end{aligned} \qquad (5)$$

The parameter δ accounts for the effect of the charge detection. In general, the magnitude of δ can have any value between a perfect detection ($\delta=0$) and no detection ($|\delta|=1$) limit. It plays a similar role as the Feynman-Vernon influence functional[2,16] which is the overlap between two states of the field that arises from the vacuum in two different trajectories of a double-slit-like experiment. In the absence of coupling to the detector (no charge detection), the two states for the detector are identical $|d_0\rangle=|d_1\rangle$. Then, the reduced density matrix is just that of the W state of the N qubits, i.e., $\varrho_N = |\Psi_G\rangle\langle\Psi_G|$. As the charge detector begins to distinguish the two charge states of the kth dot, $|\delta|$ becomes smaller than 1. As a consequence, the coherence between the state $|k\rangle$ and the other state $|W_{N-1}^{(k)}\rangle$ is partially lost. Eventually, for a perfect charge detection ($\delta=0$), the state of the composite system is given as an incoherent mixture of the two different charge states for the kth dot,

$$\varrho_N(\delta=0) = \frac{N-1}{N}|W_{N-1}^{(k)}\rangle\langle W_{N-1}^{(k)}| + \frac{1}{N}|k\rangle\langle k|. \quad (6)$$

For large N, this incoherent mixture is almost equivalent to a W state of $N-1$ quantum dots: $\varrho_N(\delta=0) \simeq |W_{N-1}^{(k)}\rangle\langle W_{N-1}^{(k)}|$. This implies that a single-electron entanglement can play an important role for the persistent current in the mesoscopic ring because the W state of $N-1$ quantum dots is a coherent state. In Secs. IV and V, we will discuss the persistent current in relation to the local charge detection and single-electron entanglement.

IV. PERSISTENT CURRENT VERSUS LOCAL CHARGE DETECTION

From the reduced density matrix $\varrho_N(\delta)$ in Eq. (5), the persistent current I_N for an N-dot ring can be evaluated as a function of the AB phase ϕ and charge detection parameter δ, as

$$I_N(\phi,\delta) = -\frac{e}{h}\mathrm{Tr}\left[\frac{\partial H}{\partial \phi}\varrho_N(\delta)\right] = \left(1-\frac{2}{N}\right)I_G(\phi) + \frac{2|\delta|}{N}I_G(\phi + \phi_\delta), \quad (7a)$$

where $I_G(\phi)$ is the persistent current of the ring in the absence of the charge detector, and $\phi_\delta=(N/2\pi)\arg(\delta)$ is an additional phase shift induced by the detector. Note that $I_G(\phi)$ is given as

$$I_G(\phi) = -\frac{e}{h}\mathrm{Tr}\left[\frac{\partial H}{\partial \phi}|\Psi_G\rangle\langle\Psi_G|\right] = I_0 \sin\left[\frac{2\pi}{N}\phi\right], \quad (7b)$$

where the oscillation amplitude is $I_0=2et/N\hbar$.

Several interesting features are found from Eq. (7). First, a finite persistent current exists even for $\phi=n\pi$ where $I_G=0$:

$$I_N(\phi=n\pi,\delta) = (-1)^n\frac{2}{N}|\delta|I_0 \sin[\arg(\delta)]. \quad (8)$$

In the absence of the charge detector, the time-reversal symmetry of the ring gives zero persistent current ($I_G=0$) for $\phi=n\pi$ that corresponds to the integer (even n) or half-integer (odd n) flux quantum. However, this is not the case if the ring is interacting with an external system. That is, persistent current may flow even at an integer or half-integer flux quantum if $|\delta|\neq 0$ with $\arg(\delta)\neq 0$. The additional phase shift induced by the detector breaks (effectively) the time-reversal symmetry of the ring and gives rise to a finite persistent current. Interestingly, even for $|\delta|=1$ the persistent current is influenced by the phase shift $\arg[\delta]$. This implies that the persistent current allows us to observe the phase variation between the states $|d_0\rangle$ and $|d_1\rangle$. Although it is not clear what kind of specific detector would display this interesting feature, it is generally true that this phenomenon happens as far as the detector can be treated as a single-quantum system. We believe that further theoretical and experimental studies are needed to illuminate this issue.

Next, we discuss the persistent current for an arbitrary ϕ in the limit of perfect local charge detection $\delta=0$. It might be expected that the persistent current disappears once a perfect local charge detection has been performed. The conventional viewpoint of decoherence is that external degrees of freedom (the charge detector in our case) "disturbs" the electron motion of the ring and would wash out the phase coherence. However, we find that *the quantum coherence of the ring does not disappear in spite of a perfect local charge detection.* This is clearly shown in the persistent current at the $\delta=0$ limit,

$$I_N(\phi,\delta=0) = \left(1-\frac{2}{N}\right)I_G(\phi). \quad (9)$$

Equation (9) shows that the persistent current for $\delta=0$ vanishes only for a double-dot ring ($N=2$). For $N=2$, the charge states for each of the quantum dots are decided by a local charge detection. Otherwise, the charge detection determines only the local charge of the kth dot. This is the origin of the quantum coherence that survives perfect charge detection and gives a finite persistent current for $N>2$. In other words, the local charge detection destroys the coherence of the ring only partially. This behavior is better understood in terms of single-electron entanglement in the number-state representation. (See below.)

V. SINGLE-ELECTRON ENTANGLEMENT AND PERSISTENT CURRENT

The behavior of the persistent current with local charge detection can be analyzed in terms of single-electron entanglement. First, let us discuss a double-dot ring $N=2$, where the persistent current disappears completely under the perfect local charge detection. The reduced density matrix of Eq. (5) is reduced to

$$\varrho_2(\delta) = \frac{1}{2}(|1\rangle\langle 1| + |2\rangle\langle 2| + \delta|2\rangle\langle 1| + \delta^*|1\rangle\langle 2|). \quad (10)$$

For an arbitrary two-qubit state, the concurrence is a direct measure of entanglement.[17] In terms of eigenvalues $\lambda_i(i\in\{1,2,3,4\})$ of the density matrix $\tilde{\varrho}_2=\varrho_2\sigma_1^y\sigma_2^y\varrho_2^*\sigma_1^y\sigma_2^y$, the concurrence is defined by $C=\max\{0,\sqrt{\lambda_1}-\sqrt{\lambda_2}-\sqrt{\lambda_3}-\sqrt{\lambda_4}\}$, where σ_i^y is a Pauli matrix, and $\lambda_1\geq\lambda_2\geq\lambda_3\geq\lambda_4$. The concurrence ranges from zero for an unentangled state to one for a maximally entangled state. For $\varrho_2(\delta)$ of Eq. (10), we find that the concurrence is given by $C(\delta)=|\delta|$. Therefore, the degree of single-electron entanglement ranges from one to zero, depending on the detection strength. Without detection ($\delta=1$), the system is in a maximally entangled state, $|\Psi_G\rangle=1/\sqrt{2}(|10\rangle+|01\rangle)$. This corresponds to the single-electron Bell state. For $\delta=0$, the state is reduced to the completely mixed state. We find that the effect of entanglement is directly related to the persistent current as

$$I_2(\delta) = C(\delta)I_G(\phi+\phi_\delta). \quad (11)$$

Apparently, the persistent current is proportional to the concurrence. This implies that the single-particle entanglement is essential in generating a persistent current. Alternatively, this property can be understood as follows. The maximally entangled state in the absence of charge detection corresponds to the maximal quantum fluctuation of the local charge numbers. These quantum fluctuations of the electron numbers are expected to give the persistent current. Local charge detection would suppress the quantum fluctuations of the number (or the degree of the number-state entanglement). A perfect local charge detection leads to the complete suppression of number fluctuations in both quantum dots. According to the number-phase uncertainty, perfect precision of the number state [for $C(\delta)=0$] implies maximum uncertainty of the phase of the state. Consequently, the persistent current disappears for this case.

For $N>2$, a perfect local charge detection does not fully suppress the persistent current. In fact, $N-1$ local charge detectors are needed to detect all the charges of the dots. In the absence of charge detection, the electron in the ring is in the single-electron W state of N quantum dots $|W_N\rangle = \Sigma_{j=1}^{N} 1/\sqrt{N}|j\rangle$. Once we observe the states of all N quantum dots by $N-1$ local charge detectors, the state of the system will be in an unentangled mixed state. [$\varrho_N(0)$ becomes a diagonal matrix.] In this case the persistent current vanishes. Hence, we can understand that the persistent current does not disappear completely by a single local charge detector since the detector distinguishes the charge state of the kth dot only.

For $N=2$ we could understand the behavior of the persistent current in terms of the concurrence. However, for $N>2$, a standard measure of the entanglement does not exist. To understand the behavior of the persistent current for $N>2$, it is helpful to introduce the dephasing factor $d_\phi = \sqrt{\mathrm{Tr}[\varrho_N^2]}$.[18] In the limit of the complete mixed state, $\rho_N^{ij} = (1/N)\delta_{ij}$ where the dephasing factor is given by $d_\phi = 1/\sqrt{N}$. That is, the dephasing factor of the system ranges from $1/\sqrt{N}$ for a fully decoherent state to 1 for a pure state. For the density matrix of Eq. (5), the dephasing factor is given as

$$d_\phi(\delta) = \sqrt{1 - \frac{2(N-1)}{N^2}(1-|\delta|)}. \qquad (12)$$

We can see that the dephasing factor for $\delta=0$, $d_\phi(0)$, is larger than the lower bound of the dephasing factor $1/\sqrt{N}$ for $N>2$. [For $N=2$, $d_\phi(0)$ has the lowest value $1/\sqrt{2}$.] This indicates that the quantum coherence of the mesoscopic ring is not completely destroyed by a local charge detection.

Now, we discuss briefly about effects of more than one local charge detection in the N quantum dots. For example, let us discuss the case of independent local detectors. This assumption allows us to extend our approach to the case of more than one local charge detector. For simplicity, let us consider two local charge detectors. As discussed above, the two local charge detectors detect the charges of only two dots. For a perfect local charge detection, the electron in the ring is in an incoherent mixture of a W state of $N-2$ quantum dots and two quantum dot states. Compared with the single detector, the quantum fluctuations of the number would be suppressed more strongly by the two local charge detections. Thus, the amplitude of persistent current is more reduced. For $N=3$, two local charge detections lead to the complete suppression of number fluctuations in three quantum dots. In this case, the persistent current disappears. If one of two charge detectors is not perfect, the persistent current survives.

VI. POSSIBLE EXPERIMENTAL REALIZATION

Persistent currents have been measured both in single isolated rings, in ensembles of isolated rings,[9] and even in a relatively complicated structure containing connected rings.[19] Furthermore, electron numbers in lateral quantum dots made of two-dimensional electron gas can be controlled down to zero.[20,21] Therefore, it is within the state-of-the-art technology to verify our prediction experimentally. In order to verify our prediction, at least three quantum dots should be constructed. The experiment would be more easily realized for a small (but larger than two) number of dots where the energy-level spacing is much larger than thermal fluctuations.

Actually, there are many decoherence sources in experimental situations for quantum dot qubits. However, a controllable detection operation on charge qubits may give rise to the qubit decoherence, showing its unique decoherence property different from other decoherence mechanisms which are not controllable.

VII. CONCLUSIONS

We have studied the influence of local charge detection on the persistent current in an Aharonov-Bohm ring composed of N-quantum dots. The local charge detection does not entirely suppress the persistent current. The nonvanishing persistent current can be understood by analyzing the single-electron entanglement in the number-state representation.

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